Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Head of the Phystech School of Applied Mathematics and Informatics
A.M. Raygorodskiy

Work program of the course (training module)

course:	Linear Algebra/Линейная алгебра
major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 1 (fall) - Grading test

Academic hours: 60 AH in total, including:
lectures: 30 AH.
seminars: 30 AH.
laboratory practical: 0 AH.

Independent work: 30 AH.

In total: 90 AH, credits in total: 2

Author of the program: N.V. Bogachev, candidate of physics and mathematical sciences, associate professor

The program was discussed at the Chair of Discrete Mathematics 05.03.2020

Annotation

What is this course about? Linear algebra is, roughly speaking, a “linear part” of the geometry of finite-dimensional spaces. It studies vector and affine spaces, linear operators and affine transformations, and Euclidean and Hermitian structures. Linear algebra has important applications in pure mathematics, computer science, physics, engineering, and many other areas. However, this subject is very interesting in itself. Its important feature is a beautiful combination of algebra and geometry. Suppose we perform some kind of transformation, such as a reflection or stretching of the space. How do geometric objects change under this transformation? And what does remain unchanged, i.e., what are the invariants of this transformation? Linear algebra provides answers to these and other similar questions.

Your final grade will be based on your test results, your homework and the final written exam.

1. Study objective

Purpose of the course

Theoretical and practical development of the main sections of linear algebra and analytical geometry. The course should provide students with an idea of the mathematical rigor of evidence. Particular attention is paid to the practical applications of linear algebra methods in various mathematical and interdisciplinary problems, to the ability to apply the apparatus of linear algebra to a wide range of problems.

Tasks of the course

acquisition by students of theoretical knowledge and practical skills in the field of vector algebra, matrix algebra;

preparing students for the study of related mathematical disciplines;

acquisition of skills in the application of analytical methods in physics and other natural sciences.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem, and develop an action plan	UC-1.1 Systematically analyze the problem situation, identify its components and the relations between them
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
Gen.Pro.C-3 Develop mathematical models and conduct their analysis in the processes of professional problem-solving	Gen.Pro.C-3.3 Gain knowledge of analytical and computational methods of problem-solving, understand the limitations for applying the obtained solutions
Pro.C-1 Become part of a professional community and conduct local research under scientific guidance using methods specific to a particular professional setting	Pro.C-1.1 Apply principles of scientific work, methods of data collection and analysis, ways of argumentation; prepare scientific reviews, publications, abstracts, and bibliographies on research topics in Russian and English
	Pro.C-1.2 Understand the verification process of software models used to solve related scientific problems

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- basic concepts of linear algebra: vector space, linear functions and operators, bilinear forms, matrix, eigenvalues and eigenvalues; - basic concepts of general algebra: groups, rings, fields;
- ideas of the coordinate method in analytic geometry;
- key theorems of linear algebra: matrix rank theorem, linear space isomorphism theorem, Jordan normal form theorem, Kronecker theorem, theorems on reducing a quadratic form and a pair of forms to canonical form.

be able to:

- solve systems of linear algebraic equations;
- perform coordinate replacement for linear and bilinear functions, linear operators;
- find the eigenvalues and eigenvectors of the linear operator;
- lead to the canonical form of a matrix of a quadratic form and a linear operator;
- solve the main problems of analytic geometry;
- prove the key theorems of linear and general algebra;
- use the apparatus of linear algebra to solve applied problems.

master:

- general concepts and definitions associated with vectors: linear independence, basis, orientation of the plane and space;
- orthogonal and affine classification of lines and surfaces of the second order.

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Vector Spaces. Linear Independence. Basis. Dimension. Linear Maps. Coordinates.	6	6		6
2	Affine Transformations and Motions.	6	6		6
3	Symmetric Bilinear Functions (Forms). Quadratic Forms. Inertial Law. Orthogonal Basis for Symmetric Bilinear Forms.	6	6		6
4	Linear Operators. Eigenspaces.	6	6		6
5	Non-Euclidean Geometry: n-sphere.	6	6		6
AH in total		30	30		30
Exam preparation		0 AH.			
Total complexity		90 AH., credits in total 2			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 1 (Fall)

1. Vector Spaces. Linear Independence. Basis. Dimension. Linear Maps. Coordinates.

Affine Spaces. Affine Hull. Euclidean Affine Geometry

2. Affine Transformations and Motions.

Bilinear Functions (Forms).

3. Symmetric Bilinear Functions (Forms). Quadratic Forms. Inertial Law. Orthogonal Basis for Symmetric Bilinear Forms.

Euclidean and Hermitian Spaces. Gram matrices. Euclidean Affine Spaces. Convex Sets and Convex Polyhedra. The Minkowski-Weyl Theorem.

4. Linear Operators. Eigenspaces.

Diagonalization of Symmetric Operators. Polar Decomposition.

5. Non-Euclidean Geometry: n-sphere.

Non-Euclidean Geometry: hyperbolic Lobachevsky space.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

A standard classroom.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

1. Линейная алгебра [Текст], учебник для вузов /В. А. Ильин, Э. Г. Позняк. М., ФИЗМАТЛИТ, 2014
2. Линейная алгебра и некоторые ее приложения [Текст] : учеб. пособие для вузов / Л. И. Головина .— 3-е изд., перераб. и доп. — М. : Наука, 1979 .— 392 с.

Additional literature

1. Аналитическая геометрия и линейная алгебра [Текст] : учеб. пособие для вузов / А. Е. Умнов ; М-во образования и науки РФ, Моск. физ.-техн. ин-т (гос. ун-т .— 3-е изд., испр. и доп .— М. : МФТИ, 2011 .— 544 с.

7. List of web resources that are necessary for the course (training module) mastering

Not used

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

Multimedia technologies can be employed during lectures and practical lessons, including presentations.

9. Guidelines for students to master the course

1. It is recommended to successfully pass the test papers, as this simplifies the final certification in the subject.
2. To prepare for the final certification in the subject, it is best to use the lecture materials.

SUPPLEMENT

Assessment funds for course (training module)

major:	Applied Mathematics and Informatics
specialization:	Advanced Methods of Modern Combinatorics/Продвинутые методы современной комбинаторики
	Phystech School of Applied Mathematics and Informatics
	Chair of Discrete Mathematics
term:	1
qualification:	Master

Semester, form of interim assessment: 1 (fall) - Grading test

Author: N.V. Bogachev, candidate of physics and mathematical sciences, associate professor

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Use a systematic approach to critically analyze a problem, and develop an action plan	UC-1.1 Systematically analyze the problem situation, identify its components and the relations between them
Gen.Pro.C-1 Address current challenges in fundamental and applied mathematics	Gen.Pro.C-1.1 Apply fundamental scientific knowledge, new scientific principles, and research methods in applied mathematics and computer science
Gen.Pro.C-3 Develop mathematical models and conduct their analysis in the processes of professional problem-solving	Gen.Pro.C-3.3 Gain knowledge of analytical and computational methods of problem-solving, understand the limitations for applying the obtained solutions
Pro.C-1 Become part of a professional community and conduct local research under scientific guidance using methods specific to a particular professional setting	Pro.C-1.1 Apply principles of scientific work, methods of data collection and analysis, ways of argumentation; prepare scientific reviews, publications, abstracts, and bibliographies on research topics in Russian and English
	Pro.C-1.2 Understand the verification process of software models used to solve related scientific problems

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- basic concepts of linear algebra: vector space, linear functions and operators, bilinear forms, matrix, eigenvalues and eigenvalues; - basic concepts of general algebra: groups, rings, fields;
- ideas of the coordinate method in analytic geometry;
- key theorems of linear algebra: matrix rank theorem, linear space isomorphism theorem, Jordan normal form theorem, Kronecker theorem, theorems on reducing a quadratic form and a pair of forms to canonical form.

be able to:

- solve systems of linear algebraic equations;
- perform coordinate replacement for linear and bilinear functions, linear operators;
- find the eigenvalues and eigenvectors of the linear operator;
- lead to the canonical form of a matrix of a quadratic form and a linear operator;
- solve the main problems of analytic geometry;
- prove the key theorems of linear and general algebra;
- use the apparatus of linear algebra to solve applied problems.

master:

- general concepts and definitions associated with vectors: linear independence, basis, orientation of the plane and space;
- orthogonal and affine classification of lines and surfaces of the second order.

3. List of typical control tasks used to evaluate knowledge and skills

Examples of tasks

1. Find the basis and dimension of the space of polynomials p (x) of degree 4 such that p (5) = 0.
2. Prove that the space of continuous functions on the segment [0,1] is infinite-dimensional.
3. Prove that if a polytope is both simple and simplicial, then it is either a simplex or a polygon.
4. Prove that the determinant of an integer skew-symmetric matrix is the square of an integer.

4. Evaluation criteria

1. Vector algebra

Vectors and actions on them. Basis, coordinates of vectors in the basis. The operations of adding vectors and multiplying them by numbers in coordinates. Linearly dependent systems of vectors. General Cartesian coordinate system, rectangular system. Scalar product, its properties, expression in the Cartesian coordinate system. Formulas for determining the distance between two points and the angle between two directions. Oriented triples of vectors. Vector product, its properties and expression in coordinates. Mixed product, its properties, expression in the general Cartesian coordinate system. Area oriented parallelogram. Oriented box volume. Collinearity and coplanarity conditions for vectors. Changing the coordinates of a vector when changing the basis.

2. The method of coordinates on the plane and in space.

Straight on the plane. Vector equation of a line, equation of a line in coordinates. Different forms of the equation of a line associated with different ways of defining it. The distance from a point to a line. Equation of a line in vector and coordinate form. Different forms of plane equation. The main tasks on the line and the plane in space.

3. Matrices. Determinants. The rank of the matrix. General theory of systems of linear equations. Gauss method.

Addition and multiplication of matrices. Inverse matrix. Definition and simplest properties of determinants. Minors, algebraic additions, decomposition of the determinant into elements of a row and column. The solution of a system of linear equations according to the Cramer rule. The rank of the matrix. Matrix rank theorem. Basis minor theorem. Systems of linear equations. Kronecker-Capelli theorem. The fundamental system of solutions and the general solution of a homogeneous system of linear equations. General solution to a heterogeneous system. Gauss method.

4. Linear space.

Linear space definition. Basis, dimension. Isomorphism theorem. Vector components in the basis, recording operations on vectors through components. Change in the components of the vector when the basis changes. Transition matrix. Subspaces in linear space. Sum and intersection of subspaces. Direct amount.

5. Linear operators.

Linear operators, their matrices. Operations on linear operators, inverse linear operator. A change in the matrix of a linear operator when the basis changes. Eigenvectors and eigenvalues of linear operators. Invariant subspaces. Finding eigenvectors and eigenvalues of a linear operator. The characteristic equation. Linear independence of eigenvectors belonging to different eigenvalues. Own spaces. Jordan's normal form.

6. Quadratic forms

Bilinear and quadratic forms. Reduction of a quadratic form to canonical form. The law of inertia of quadratic forms. Positive definite quadratic forms, Sylvester criterion.

7. Euclidean space. Hermitian space, adjoint operators

Euclidean space. Hermitian space. Scalar product. Gram matrix. Orthogonal normalized basis, orthogonalization process. Orthogonal complement of subspace. Orthogonal matrices. Conjugate operators, their properties. Self-adjoint operators, properties of their eigenvectors and eigenvalues. Existence of an orthonormal basis of eigenvectors of a self-adjoint operator. Polar expansion of a linear operator in Euclidean space. Pseudo-Euclidean space. Lorentz transformations.

8. Algebra of polynomials

Properties of the roots of polynomials, formal derivative, roots of polynomials with real coefficients, theory of divisibility in a Euclidean ring, polynomials with rational coefficients, symmetric polynomials.

9. Groups, rings, fields

The concept of a group, finite groups, Lagrange's theorem, Cayley's theorem. Abelian groups, a theorem on the structure of finitely generated abelian groups. The concept of the ring, the concept of the field. Field of deductions, final fields. Vector space over an arbitrary field, examples.

10. Applications of linear algebra:

Numerical methods for solving a system of linear equations. Matrices of small rank, skeletal decomposition. Principal component method, projectors and least squares method. Markov matrices. Differential equations, matrix exponent.

11. Tensors

The concept of tensors. Vectors and covectors, tensor rank. Tensor product, convolution. Tensor in space with a metric, raising and lowering the index. Examples of tensor quantities in physics.

12. Lines and surfaces of the second order

A theorem on reducing the equation of a second-order line on a plane to canonical form. Ellipse, hyperbola and parabola. Theorems on tricks and directrixes. Asymptotes of hyperbole. Tangent to ellipse, hyperbole and parabola. Surface rotation. Ellipsoid, paraboloids, hyperboloids. Canonical equations and appearance.

Assessment “excellent (10)” is given to a student who has displayed comprehensive, systematic and deep knowledge of the educational program material, has independently performed all the tasks stipulated by the program, has deeply studied the basic and additional literature recommended by the program, has been actively working in the classroom, and understands the basic scientific concepts on studied discipline, who showed creativity and scientific approach in understanding and presenting educational program material, whose answer is characterized by using rich and adequate terms, and by the consistent and logical presentation of the material;

Assessment “excellent (9)” is given to a student who has displayed comprehensive, systematic knowledge of the educational program material, has independently performed all the tasks provided by the program, has deeply mastered the basic literature and is familiar with the additional literature recommended by the program, has been actively working in the classroom, has shown the systematic nature of knowledge on discipline sufficient for further study, as well as the ability to amplify it on one’s own, whose answer is distinguished by the accuracy of the terms used, and the presentation of the material in it is consistent and logical;

Assessment “excellent (8)” is given to a student who has displayed complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently performed all the tasks stipulated by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (7)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in the answer, has independently performed all the tasks provided by the program, studied the basic literature recommended by the program, worked actively in the classroom, showed systematic character of his knowledge of the discipline, which is sufficient for further study, as well as the ability to amplify it on his own;

Assessment “good (6)” is given to a student who has displayed a sufficiently complete knowledge of the educational program material, does not allow significant inaccuracies in his answer, has independently carried out the main tasks stipulated by the program, studied the basic literature recommended by the program, showed systematic character of his knowledge of the discipline, which is sufficient for further study;

Assessment “good (5)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, mastered the basic literature recommended by the program, made some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors by himself;

Assessment “satisfactory (4)” is given to a student who has discovered knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, who while not being sufficiently active in the classroom, has nevertheless independently carried out the main tasks stipulated by the program, learned the main literature but allowed some errors in their implementation and in his answer during the test, but has the necessary knowledge for correcting these errors under the guidance of a teacher;

Assessment “satisfactory (3)” is given to a student who has displayed knowledge of the basic educational program material in the amount necessary for further study and future work in the profession, not showed activity in the classroom, independently fulfilled the main tasks envisaged by the program, but allowed errors in their implementation and in the answer during the test, but possessing necessary knowledge for elimination under the guidance of the teacher of the most essential errors;

Assessment “unsatisfactory (2)” is given to a student who showed gaps in knowledge or lack of knowledge on a significant part of the basic educational program material, who has not performed independently the main tasks demanded by the program, made fundamental errors in the fulfillment of the tasks stipulated by the program, who is not able to continue his studies or start professional activities without additional training in the discipline in question;

Assessment “unsatisfactory (1)” is given to a student when there is no answer (refusal to answer), or when the submitted answer does not correspond at all to the essence of the questions contained in the task.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

When conducting a differential test, the student is given at least 60 minutes to prepare. A student’s survey on the differential test should not exceed two astronomical hours.

During the differential test, students can use the discipline program.